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Extra dimensions and the strong CP problem

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Abstract

In higher-dimensional theories such as Brane World models with quasi-localized non-Abelian gauge fields the vacuum structure turns out to be trivial. Since the gauge theory behaves at large distances as a $4+\delta$-dimensional and thus the topology of the infinity is that of of $S^{3+\delta}$ rather than S^3, the set of gauge mappings are homotopically trivial and the CP-violating θ-term vanishes on the brane world-volume. As well there are no contributions to the θ-term from the higher-dimensional solitonic configurations. In this way, the strong CP problem is absent in the models with quasi-localized gluons.

Despite the fact that the Standard Model (SM) of elementary particles and their interactions is extremely successful in describing almost all currently available experimental data, there are some puzzling theoretical points which force to think that a certain more fundamental physics stay behind the SM. One of such puzzles is the strong CP problem. It is reflected in the appearance of CP-violating θ-term in the effective Lagrangian of QCD [1]:

$$\mathcal{L}_\theta = \frac{\theta}{32\pi^2} Tr\left(G^a_{\mu\nu}\widetilde{G}^{a\mu\nu}\right), \tag{1}$$

where $G^a_{\mu\nu}$ and $\widetilde{G}^{a\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}G^a_{\rho\sigma}$ are gluon field strength and its dual, respectively. The experimental data on the electric dipole moment of neutron constraints θ to be extremely small[1], $\theta \lesssim 10^{-10}$, although a priori there are no theoretical reasons why it should be so tiny.

The existence of the θ-term (1) in the effective QCD Lagrangian is related to the multiple structure of the QCD vacuum. Although (1) can be expressed as a full derivative it nevertheless does not vanishe at infinity because of the presence of topologically non-trivial instanton gauge field configurations [2]. Instantons, being stable, finite action solutions to the Euclidean (non-Abelian) gauge field equations of motion, are interpreted physically as a quantum-mechanical tunnellings between the different vacuum states $|n>$ which are characterized by a topological index $n \in Z$ [3]. The true vacuum of the theory is a superposition of these vacuum states:

$$|\theta> = \sum_{n=-\infty}^{n=\infty} e^{in\theta}\,|n> . \tag{2}$$

The stability of the instantonic configurations is guaranteed by the non-trivial topology. Indeed, any vacuum configuration of a vector field A_μ corresponding to the gauge group G has the form $A_\mu = U^{-1}\partial_\mu U$, where U is an element of G. Actually, U defines a map from the geometry of a spatial infinity I to a gauge group G:

$$U : I \to G. \tag{3}$$

In four dimensions I is isomorphic to a 3-sphere S^3 and if G is a gauge group which contains an $SU(2)$ as a subgroup, the mapping (3) fall into the homotopically non-equivalent classes each characterized by the topological winding number $n \in \pi_3\left(SU(2)\right) = Z$. The conservation of the topological charge prevents the instantons from decaying into the trivial vacuum configuration. This is actually the case for the colour $SU(3)$ group of QCD. Thus, the strong CP problem emerges due to the existence of the $|n>$-vacua structure in QCD and due to the non-vanishing amplitude of tunnelling between different $|n>$-vacua. Consequently, the solution to the strong CP problem can be achieved either by suppressing the tunnelling or making the $|n>$-vacua unstable. Most of the existing solutions to the

[1]More precisely, this bound applies to the effective θ-term defined as $\overline{\theta} = \theta + agr\det\widehat{M}$, where $\widehat{M}$ is the quark mass matrix.

strong CP problem rely on the first possibility. Perhaps the most elegant among them is the familiar Peccei-Quinn mechanism [4] which actually forbids tunnellings due to an extra anomalous $U(1)_{PQ}$ setting $\overline{\theta} = 0$ dynamically. To realize the second possibility one should change the global geometry of space-time in such a way that mapping (3) becomes homotopically trivial. In this letter we discuss such a possibility in a particular model with extra dimensions[2].

One can find plenty of purely phenomenomenological motivations why the extra dimensions beyond those four observed so far could actually be present in nature. Moreover, a consistent unification of all fundamental forces within the string theory seems also to require introduction of extra dimensions. Recently, the idea of extra dimensions received a new twist after the observation that their effects might be accessible to probe in experiments in the near future, providing the size of extra dimensions are large enough ($\lesssim$TeV^{-1}) [6, 7] or the extra space is warped [8, 9]. The common ingredient of all these theoretical constructions, often called as a Brane World scenario, is a (3+1)-dimensional hypersurface (3-brane) embedded in higher dimensional space-time, where the SM fields [6] and perhaps gravity as well [8] are confined. Thus the crucial theoretical question is how the fields of various spin are localized on a 3-brane (for earlier works see, e.g. [10]). There are significant difficulties in localizing massless higher-spin fields, and in particular gauge fields, on a 3-brane [10]. In this respect the approach proposed in a series of recent papers [11, 12] is very attractive since it can be universally applied to all kind of fields.

The basic idea behind this approach is rather transparent [11, 12]. Consider some field freely propagating in the higher-dimensional space-time (bulk) and having the coupling with another field which is localized on the 3-brane. In most general situations one can expect that the radiative corrections with localized field running in the loops can induce the non-trivial terms (including the kinetic one) for the bulk field on the 3-brane world-volume. Then the emerging physical picture for the bulk field is the following: At small distances measured on the 3-brane world-volume the induced 4-dimensional kinetic term dominates over the higher-dimensional one and the bulk field essentially behaves as a 4-dimensional one. At large distances, however, the original higher-dimensional kinetic term becomes dominant and the field behaves as a higher-dimensional. The crossover scale is actually controlled by the ratio of the parameters in the original kinetic term and in that of the induced one (that is the ratio of higher-dimensional and induced gauge couplings in the case of gauge fields, for example) and should be adjusted in order not to contradict the already known experimental data. This quasi-localization mechanism has been successfully applied to the gravity [11] and gauge interactions [12] and, as we mentioned above, can be used for other fields (fermions and bosons) as well. The remarkable thing offered by this mechanism is that the extra space-time now can be truly infinite, unlike the case of the ordinary Kaluza-Klein or warped compactification. This fact can be further explored as a crucial standpoint for the solution of the cosmological constant problem [11]. Here we would like to point out another feature of the above scenario with

[2]For an earlier proposal within a different model see [5].

quasi-localized gauge fields which concerns the strong CP problem. The idea is due to the above-mentioned observation that the existence of the θ-term (1) is essentially related to the global geometry of space-time. In particular, since the gauge theory in the scenario of ref. [12] become $(4+\delta)$-dimensional at large distances and the topology of spatial infinity is that of $S^{3+\delta}$ rather than S^3, the set of gauge mappings (3) become homotopically trivial and the θ-term (1) disappears from the 3-brane world-volume.

To be more quantitative let us begin by considering a pure $SU(2)$ gauge theory which lives in the 5-dimensional bulk. It is defined by the covariant derivative $D_M = \partial_M + iA^a_M\sigma^a/2$ with field strengths $G_{MN} = G^a_{MN}\sigma^a/2 = -i\,[D_M, D_N]$, where the capital letters run over the bulk coordinates, $M(N) = (\mu(\nu) = 0, 1, 2, 3; y)$, while those of small, $a, b = 1, 2, 3$ are $SU(2)$-adjoint indices and σ^a are the Pauli matrices. We assume also that a (3+1)-dimensional δ-like brane is embedded in 5-dimensional bulk space-time with a certain matter fields charged under the $SU(2)$ symmetry are localized on it. Then the radiative corrections involving these matter fields induce the (3+1)-dimensional kinetic term for the $SU(2)$ gauge field on the 3-brane world-volume, so that the total effective Lagrangian can be written as:

$$\mathcal{L} = -\frac{1}{2g_5^2}G_{MN}G^{MN} - \frac{\delta(y)}{2g_4^2}G_{\mu\nu}G^{\mu\nu} + ..., \tag{4}$$

where g_5 is the 5-dimensional gauge coupling with mass dimension -1/2 and g_4 is the dimensionless 4-dimensional gauge coupling and $G_{\mu\nu}(x^\mu) = G_{MN}(x^\mu, y = 0)\delta^M_\mu\delta^N_\nu$. Following [12], we define the crossover scale r_c as:

$$r_c = \frac{g_5^2}{2g_4^2}, \tag{5}$$

which need not be extremely large, i.e. of the order of the present Hubble size, as it could be naively expected, but even $r_c \sim 10^{15}$cm (the solar system size) can be compatible with the existing observations due to the phenomenon of infrared transparency (see [12] for more details).

Let us first consider two limiting cases: $r_c \to 0$ and $r_c \to \infty$. When $r_c \to 0$ ($g_4 \to \infty$, $g_5 = const$), the induced kinetic term in (4) is completely irrelevant and thus the $SU(2)$ gauge theory behaves as an exactly 5-dimensional one. Let us split coordinates as $x^M = (x^0, x^\alpha)$, where $x^\alpha = (x^i, y)$ and $i = 1, 2, 3$ runs over the spatial 3-brane world-volume coordinates. Now, we can use the well-known fact of formal mathematical equivalence between the static solitons in D spatial dimensions and the instantons in $D - 1$ space and one imaginary-time dimensions. So, to construct the static solitonic solutions in 5 dimensions we must just change the imaginary time coordinate $\widetilde{x}^0 = ix^0$ in the original instanton configuration of the Euclidean 4-dimensional theory by the fifth coordinate y. Thus, the soliton in five dimensions can be written in the form:

$$\begin{aligned} A_0 &= 0, \\ A_\alpha &= -\frac{i}{g_5}\eta_{a\alpha\beta}\sigma^a\partial_\beta \ln U, \end{aligned} \tag{6}$$

where

$$U(x^\alpha) = \rho^2 + \lambda^2, \tag{7}$$

$\rho^2 = \left(x^i - \xi^i\right)^2 + \left(y - \xi^y\right)^2$, λ is the size of the soliton and $\eta_{a\alpha\beta}$ is the 't Hooft symbol:

$$\eta_{a\alpha\beta} = \delta_{a\alpha}\delta_{y\beta} - \delta_{a\beta}\delta_{y\alpha} + \epsilon_{a\alpha\beta} \tag{8}$$

The above configuration describes a static topologically stable soliton centered at $x^\alpha = \xi^\alpha$ with energy $E = 8\pi^2/g_5^2$ and thus represents the nontrivial homotopy $\pi_3\left(SU(2)\right)$ (just like the instanton in four dimensions). The configuration (6),(7) satisfy the self-duality equation of the form

$$\widetilde{G}_{0\alpha\beta} = G_{\alpha\beta}, \tag{9}$$

where $\widetilde{G}_{ABC} = \epsilon_{ABCDE}G^{DE}$. The topological charge q is determined through the 4-volume integral over the time component of the conserved current

$$Q^A = \frac{g_5^2}{16\pi^2} Tr \widetilde{G}^{ABC} G_{BC} \tag{10}$$

which for (6),(7) is equal to one:

$$q = \int \prod_{i=1}^{3} dx^i dy Q^0 = \frac{g_5^2}{8\pi} E = 1. \tag{11}$$

In the opposite extreme case, i.e. when $r_c \to \infty$ ($g_5 \to \infty$, $g_4 = const$), the first term in (4) disappears and theory becomes (3+1)-dimensional. Passing to the Euclidean coordinates ($x^0 \to ix^0$) we come back to the ordinary instanton configuration located on the 3-brane world-volume at $y = 0$ by changing y to x^0 in (6),(7) with $A_y^a = 0$ instead of $A_0^a = 0$ and $\mu = 0, 1, 2, 3$ instead of $\alpha = 1, 2, 3, y$.

In the case of finite non-zero r_c the gauge fields behave as (3+1)-dimensional at distances $|x^i| \ll r_c$ and as (4+1)-dimensional at large distances $|x^i| \gg r_c$. Hence, the spatial infinity is a 4-sphere S^4 rather than S^3. Because of this, the (3+1)-dimensional instanton is not stable against dispersion since the gauge mappings $S^4 \to SU(2)$ is trivial. That is to say, the boundary behavior of $SU(2)$ gauge bosons can be always continuously deformed to the global (trivial) vacuum configuration while keeping the action finite and therefore the instantons decay into this vacuum. Thus all the gauge freedom can be removed by gauge fixing and $|n>$-vacua and the θ-term, consequently, do not arise.

The case of the soliton solution (6),(7) is somewhat different. In the case of finite r_c the boundary conditions on the 3-brane located at $y = 0$ become relevant. Since the translational invariance along the fifth coordinate is broken by the presence of a 3-brane, the Neumann boundary condition,

$$\left.\frac{\partial A_\mu}{\partial y}\right|_{y=0} = 0, \tag{12}$$

is satisfied by the one-solitonic configuration (6),(7) only when $\xi^y = 0$. One can also construct the multiinstanton-like solitonic solution with

$$U = \left(\frac{\lambda^2}{\rho_+^2} + \frac{\lambda^2}{\rho_-^2} + 1\right) \qquad \rho_\pm^2 = \left(x^i\right)^2 + (y \mp \xi)^2 \tag{13}$$

in (6) instead of (7) (see also [13]). This solution describes the static soliton centered at $(0,0,0,0,\xi)$ and its image at $(0,0,0,0,-\xi)$ and also satisfies the boundary condition (12). Now, if the fermionic matter (quarks) are localized on the 3-brane, the θ-term on the brane will have the form:

$$\mathcal{L}_\theta = \frac{\theta}{2r_c} \int dy \delta(y) Q^y. \tag{14}$$

It is easy to see that the above solitonic configurations indeed lead to the vanishing of (14) simply because of $Q^y = 0$.

Let us stress that the absence of θ-term is a feature of the particular models with quasi-localized gauge fields of ref. [12]. Nothing similar happen in more conventional models with compactified dimensions where the space-time geometry is a direct product of $M^4 \times C^N$ (M^4 is a 4-dimensional Minkowski space-time and C^N is an N-dimensional compactified internal space). Here we have actually two mappings: $S^3 \to SU(2)$ and $B \to SU(2)$, where S^3 is a boundary of a Euclideainized M^4 while B is a boundary of C^N. In fact both mappings might be non-trivial. Thus at least instantons (and θ-term, consequently) will remain in M^4 although there might exist other stable topological defects depending on the geometry of extra space-time C^N [14].

To conclude, we showed that the QCD vacuum is indeed trivial in a certain class of Brane World models with quasi-localized gluons and thus CP-violating θ-term is absent. However, in solving the strong CP problem in this way, we are loosing the instantonic solution to the $U(1)_A$ problem (i.e. the origin of the $\eta^{'}$ mass). On the other hand, in the naive quark model there is a simple and natural explanation of the fact why the $\eta^{'}$ is much heavier than the pion [15]: in the $\eta^{'}$, which is an isosinglet, the quark - antiquark pair can annihilate into gluons while such an annihilation is absent for the isovector π's. Moreover, as it was argued by Witten and Veneziano [16] long ago the instantonic dynamics underlying $\eta^{'}$ mass is in conflict with predictions based on the large N_c expansion. They have suggested that the true dynamical origin of the $\eta^{'}$ mass would be the coupling of the $U(1)_A$ axial anomaly to the topological charge associated with confinement-related vacuum fluctuations rather than instantons. Alternatively, it seems promising to look for intrinsically higher-dimensional solution of the $U(1)_A$ problem within the models of quasi-localized gauge fields.

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